Z(3)-symmetric effective theory for pure gauge QCD at high temperature

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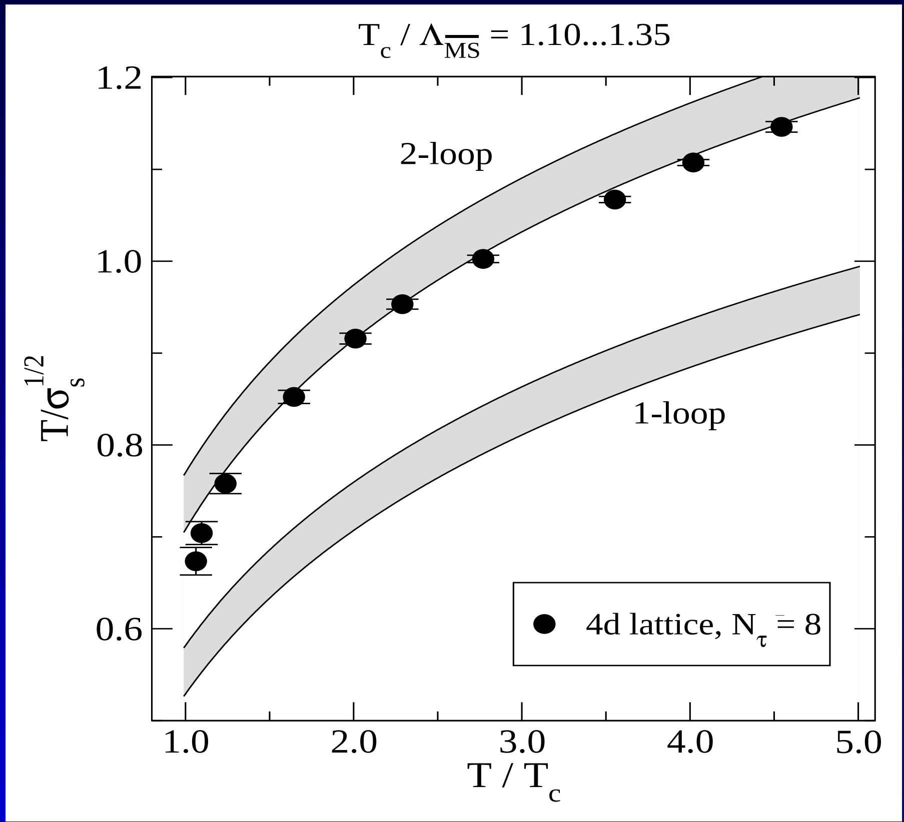
QCD and dimensional reduction

- Conventional DR: at high $T \gg gT$, integrate out all non-static modes ($m \sim 2\pi T$) to obtain $3d$ effective theory for the static modes

$$\begin{aligned}\mathcal{L}_{\text{EQCD}} &= g_3^{-2}\Big\{\tfrac{1}{2}\,\mathrm{Tr}\,F_{ij}^2 + \mathrm{Tr}\big[(D_iA_0)^2\big] \\ &+ m_{\text{E}}^2\,\mathrm{Tr}\,(A_0^2) + \lambda_{\text{E}}\,\mathrm{Tr}\,(A_0^4)\Big\} + \delta\mathcal{L}_{\text{E}}, \\ & g_3 \equiv \sqrt{T}g,\ m_{\text{E}} \sim gT,\ \lambda_{\text{E}} \sim g^2\end{aligned}$$

- New theory sufficient to describe physics of length scales $\gtrsim 1/(gT)$
- Parameters available through comparison of long distance correlators in EQCD and full QCD

- Partial success: impressive perturbative results derived at high T



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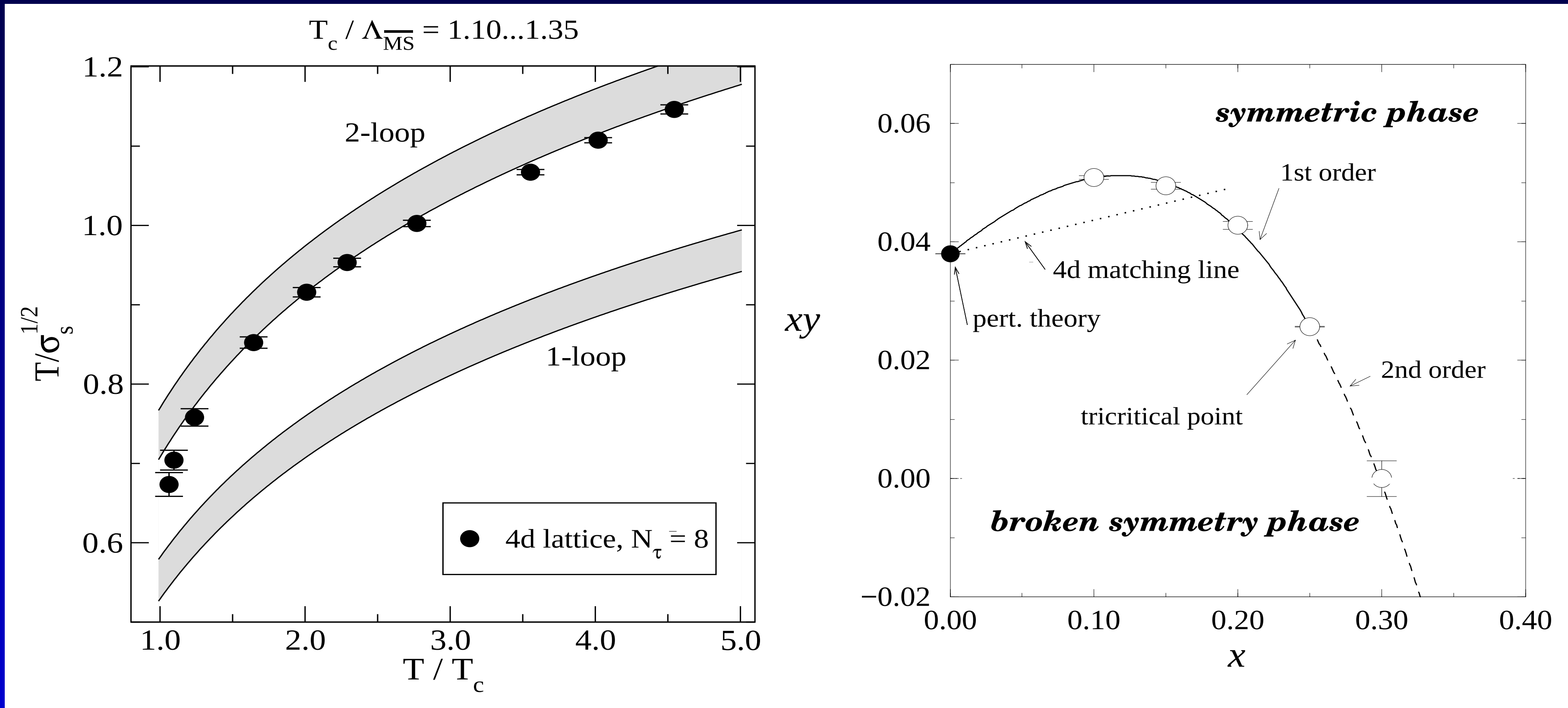

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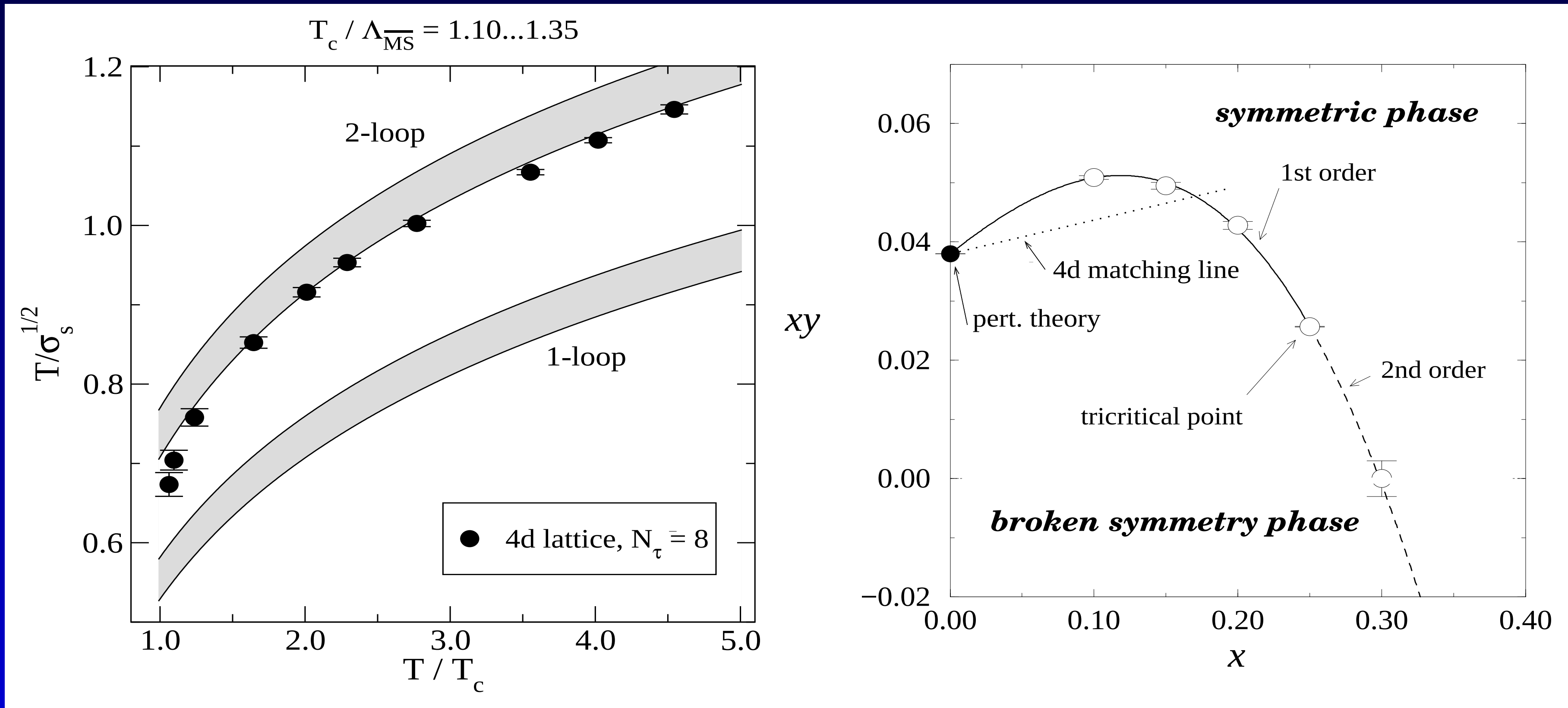

- Fundamental problem: all symmetries of original theory are *not* respected by the reduction!

- Four-dimensional SU(3) YM theory invariant under twisted gauge transformations

$$A_\mu(x) \longrightarrow s(x)\,(A_\mu(x) + i\,\partial_\mu)\,s(x)^\dagger, s(x) \in SU(3)$$

$$s(x+\beta\,\hat{e}_t) = z\,s(x)\,, z \in Z(3)$$

under which Wilson line is a Z(3) fundamental

$$\begin{aligned} \Omega(\mathbf{x}) &\equiv \mathcal{P}\,\exp\left[i\int_0^\beta \mathrm{d}\tau\,A_0(\tau,\mathbf{x})\right] \\ \mathrm{Tr}\;\Omega(\mathbf{x}) &\rightarrow z\,\mathrm{Tr}\;\Omega(\mathbf{x}) \end{aligned}$$

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- In deconfined phase, effective potential for Ω has degenerate minima $\Omega_{\text{min}} = e^{i2\pi n/3}\mathbb{1}$, $n \in \{0, 1, 2\}$
  - Tunnelings between different vacua important
  - At (1st order) phase transition quadruple point with phase coexistence with the confining one

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- EQCD Lagrangian derived expanding A_0 as a small fluctuation around $\Omega_{\mathrm{min}} = \mathbb{1}$
  - Z(3) invariance lost
  - Complex Z(3) minima $A_0 = \frac{2\pi T}{3}$ completely outside the domain of validity of eff. theory

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Sigma Models	
Non-linear	Linear
$\bar{\phi} \cdot \bar{\phi} = \mathbb{1}$	Polynomial V
Same long distance physics!	

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- Minimal set of dof's: A_i and Ω
  - $\Omega \in \mathrm{SU}(3) \Rightarrow$ polynomial Lagrangian non-renormalizable
- New (old) idea: replace Ω by $\mathcal{Z} \in$ GL(3,$\mathbb{C}$)
  - Coarse-grained version of Ω
  - After gauge fixing, contains $10 - 2 = 8$ unphysical dof's that are chosen heavier ($m \sim T$) than the physical ones ($m \lesssim gT$)

- Require gauge and Z(3) invariance

$$
\begin{aligned}
\mathcal{Z}(\mathbf{x}) &\rightarrow s(\mathbf{x})\,\mathcal{Z}(\mathbf{x})\,s(\mathbf{x})^\dagger, \\
\mathbf{A}(\mathbf{x}) &\rightarrow s(\mathbf{x})\,(\mathbf{A}(\mathbf{x}) + i\nabla)\,s(\mathbf{x})^\dagger, \\
\mathcal{Z}(\mathbf{x}) &\rightarrow \mathrm{e}^{2\pi i n/3}\,\mathcal{Z}(\mathbf{x})
\end{aligned}
$$

and compose Lagrangian as

$$
\mathcal{L} = g_3^{-2}\left\{\tfrac{1}{2}\,\mathrm{Tr}\,F_{ij}^2 + \mathrm{Tr}\left(D_i\mathcal{Z}^\dagger D_i\mathcal{Z}\right) + V(\mathcal{Z})\right\},
$$

$$
V(\mathcal{Z}) = V_0(\mathcal{Z}) + g_3^2\,V_1(\mathcal{Z})
$$

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\begin{aligned}
V(\mathcal{Z}) &= \boxed{V_0(\mathcal{Z})} + g_3^2\, V_1(\mathcal{Z}) \\
V_0(\mathcal{Z}) &= c_1\, \mathrm{Tr}\big[\mathcal{Z}^\dagger \mathcal{Z}\big] + c_2 \left(\det[\mathcal{Z}] + \det\big[\mathcal{Z}^\dagger\big]\right) \\
&+ \; c_3\, \mathrm{Tr}\left[(\mathcal{Z}^\dagger \mathcal{Z})^2\right]
\end{aligned}
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- Gives heavy fields their masses
- With $c_2 < 0 < c_3$ and $c_2^2 > 9c_1c_3$, V_0 minimized by $Z = \frac{1}{3}v\Omega$ with $\Omega \in$ SU(3) and
$$v \equiv \frac{3}{4}\left(\frac{-c_2 + \sqrt{c_2^2 - 8c_1c_3}}{c_3}\right)$$
- Invariant under extra SU(3)×SU(3) symmetry $\mathcal{Z}(\mathbf{x}) \to A\mathcal{Z}(\mathbf{x})B,\;\; A, B \in$ SU(3)

$$
\begin{aligned}
V(\mathcal{Z}) &= V_0(\mathcal{Z}) + \boxed{g_3^2\, V_1(\mathcal{Z})} \\
V_1(\mathcal{Z}) &= \tilde{c}_1\, \mathrm{Tr}\big[M^\dagger M\big] + \tilde{c}_2 \left(\mathrm{Tr}\left[M^3\right] + \mathrm{Tr}\left[(M^\dagger)^3\right]\right) \\
&+ \tilde{c}_3\, \mathrm{Tr}\big[(M^\dagger M)^2\big], \\
M &\equiv \mathcal{Z} - \tfrac{1}{3}\mathrm{Tr}\, \mathcal{Z}\mathbb{1} \equiv \mathcal{Z} - \tfrac{1}{3} L\mathbb{1}
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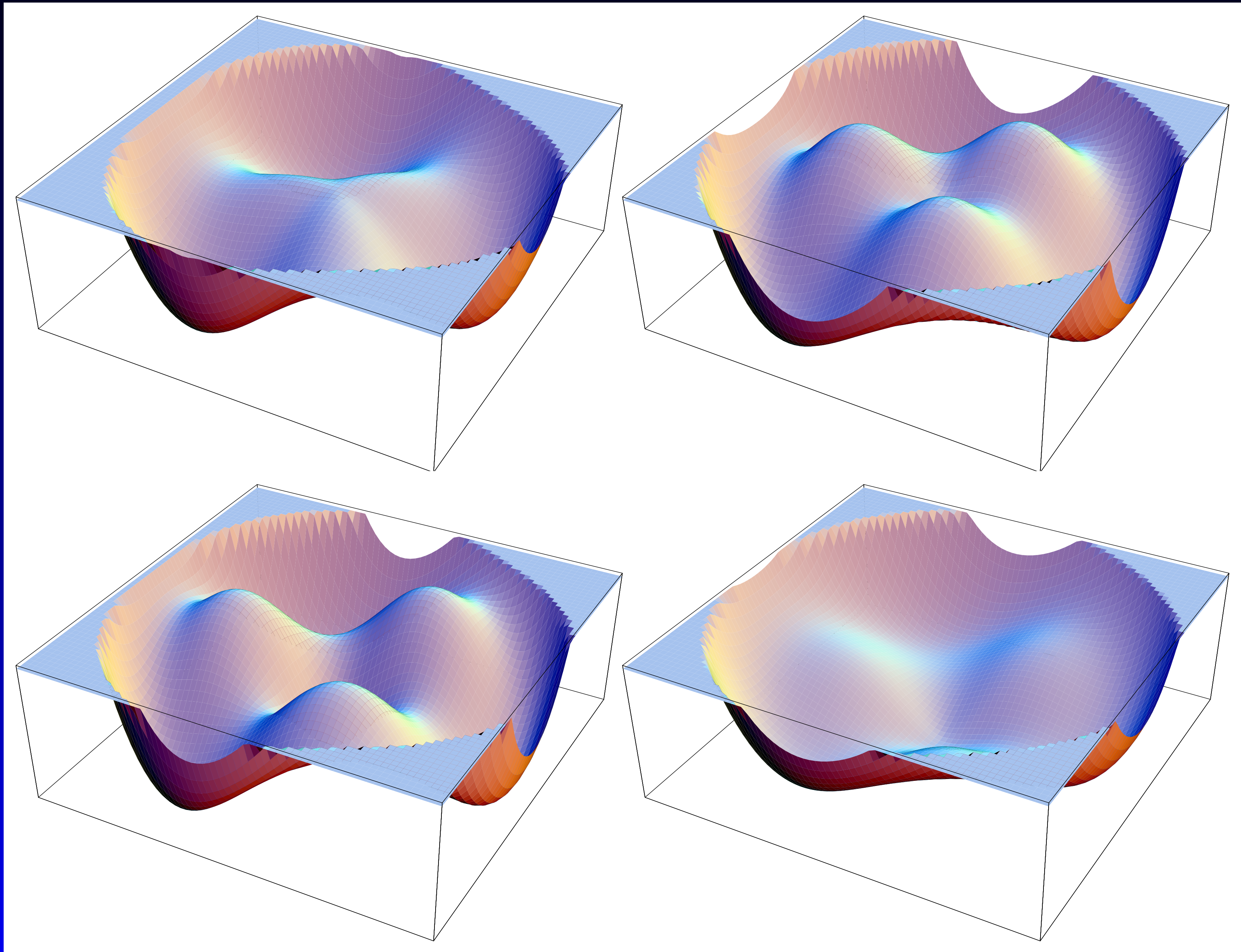
- Vital for high-T matching to EQCD
- Assuming $\tilde{c}_3 > 0$ and $\tilde{c}_2^2 < \tilde{c}_1\tilde{c}_3$, V_1 minimized by $M = 0$, *i.e.* $\mathcal{Z} = \frac{1}{3}\, L(\mathbf{x})\mathbb{1}$
- $V(\mathcal{Z})$ minimized by
  - $c_2^2 > 9c_1c_3$: $\mathcal{Z} = \frac{v}{3}\, e^{2\pi i n/3}\mathbb{1}$
  - $c_2^2 < 9c_1c_3$: $\mathcal{Z} = 0$



Matching to EQCD

- At high T and small g, consider fluctuations of $\mathcal{Z}$ around Z(3) minima

$$\mathcal{Z} \;=\; e^{2\pi i n/3}\left\{\tfrac{1}{3}v\,\mathbb{1} + g_3\left[\frac{1}{\sqrt{6}}\,(\phi + i\chi)\mathbb{1} + (h + ia)\right]\right\}$$

and demand the integration-out of ϕ, χ and h reproduces EQCD

$$\mathcal{L}_{\rm light} \;=\; \tfrac{1}{2}\,{\rm Tr}\,F_{ij}^2 + {\rm Tr}[(D_i\,a)^2 + m_a^2\,a^2 + \tilde{\lambda}\,a^4]$$

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- Immediate result:

$$c_1 \;=\; \tfrac{1}{6}(m_\chi^2 - 3m_\phi^2)\,,\;\; c_2 = -m_\chi^2/v\,,$$

$$c_3 \;=\; \tfrac{3}{4}(m_\chi^2 + 3m_\phi^2)/v^2\,,\;\; m_h^2 = m_\chi^2 + m_\phi^2$$

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- Noticing that SU(3)×SU(3) invariance guarantees V_0 does not contribute, get also:

$$\tilde{c}_1 \;=\; T + \mathcal{O}(g_3^2),\;\; \tilde{c}_3 \;=\; \frac{3}{4\pi^2 T} + \mathcal{O}\left(\frac{g_3^2}{T^2}\right)$$

Z(3) domain walls

- To capture Z(3) physics of the full theory, demand the effective one reproduce leading order domain wall properties

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- In effective theory, end up minimizing an energy functional expressed in terms of the phases of the eigenvalues of $\mathcal{Z}$

$$F_{\rm dw}[\alpha,\beta] \equiv F_{\rm grad} + F_{\rm soft} + F_{\rm fluc} =$$

$$g_3^{-1}\,(\pi\bar{v}\,T)^2\,(\tfrac{2}{3}\sqrt{T})^3 \int_{-\infty}^{\infty} d\bar{z}\,\left[(\alpha')^2 + 3(\beta')^2 + U_1 + U_2\right]$$

with $\bar{z} \equiv g_3\sqrt{T}z$, $\bar{v} \equiv \frac{v}{T}$

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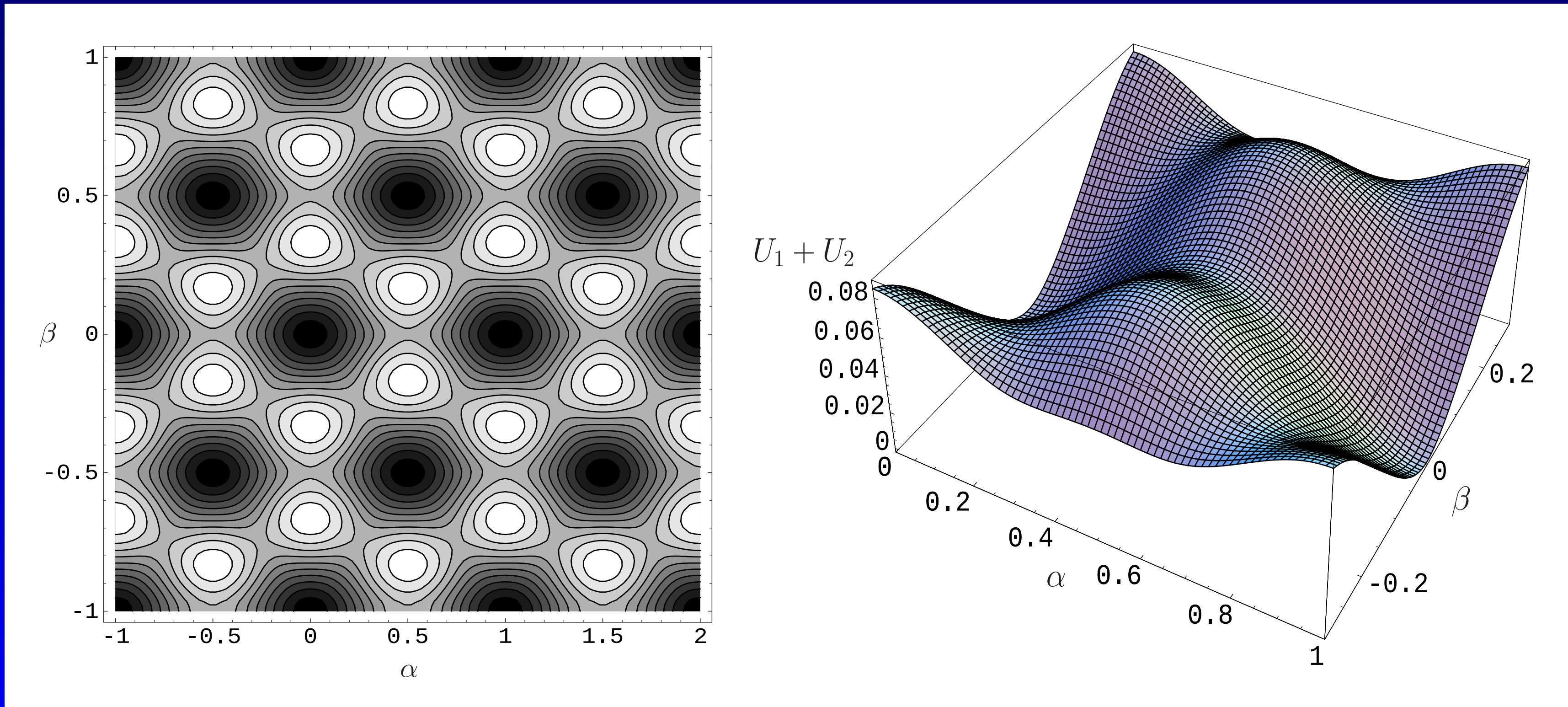

- Solve for α, β demanding that domain wall tension and width agree with full theory values
  - $\sigma_{\rm YM} = \frac{8\pi^2}{9}\,\frac{T^3}{g(T)}\,, \Delta z_{\rm YM} = \frac{\ln(4)-1/2}{g(T)\,T}$

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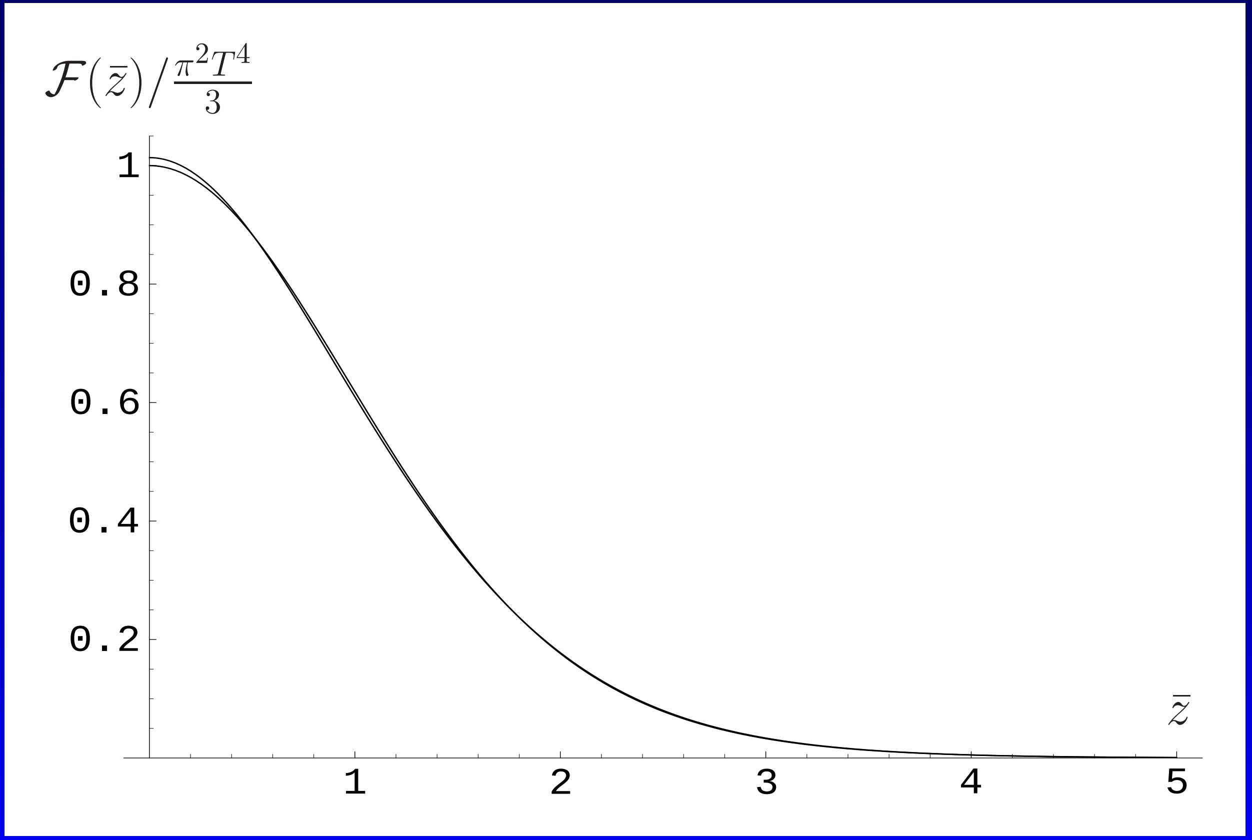
- Result: $v/T = 3.005868,\ \tilde{c}_2 = 0.118914$



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- Without perturbative matching, phase diagram parametrized by 6 dimensionless constants
    - With matching, overall scale v known and only g_3^2/v, m_ϕ/m_χ and m_ϕ/v remain

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- In full theory, phase transition known to be weakly 1st order $\Rightarrow$ latter scenario favored

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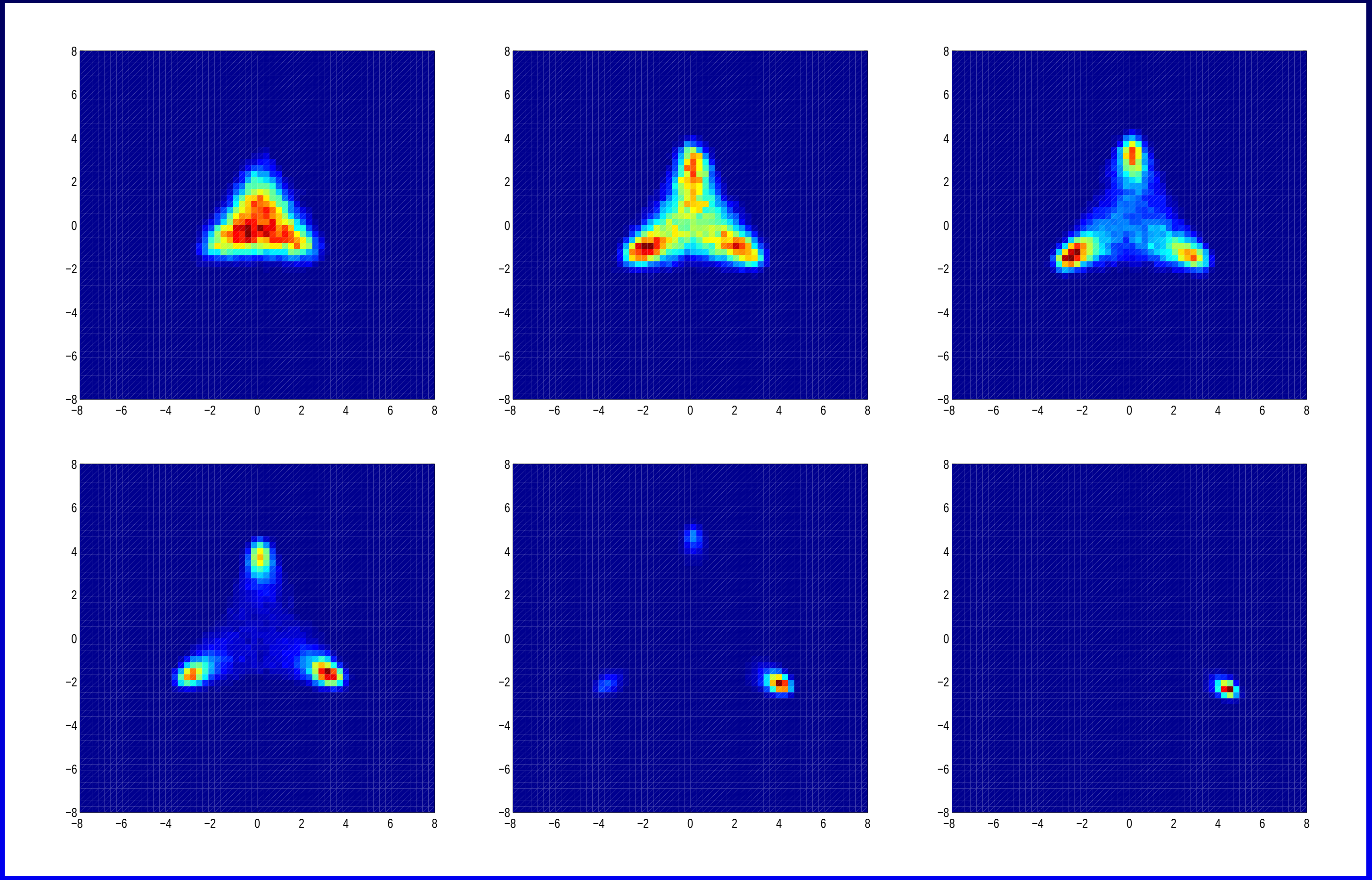
  - Match correlation lengths in various channels
  - Simulations underway (Kajantie, Kurkela)



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Disclaimer: No diagrams were calculated during the writing of this paper